

## Long-range correlations in nonequilibrium Lorentz gases

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Spatial correlation functions in Lorentz gases maintained in nonequilibrium steady states are studied. The correlation function of microscopic number densities of the moving particle and scatterer is found to have long range spatial correlations that decay as  $1/r^2$  in three dimensions in the presence of either a concentration gradient, a temperature gradient, or an electric field. Stronger correlations are found at higher order in the electric field. Our analysis is based on a derived kinetic equation for the two-particle correlation function. Possible extensions of our work are discussed.

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### I. INTRODUCTION

In this paper, the fluctuations in a classical Lorentz gas that is in nonequilibrium steady states are studied. In particular, we study the spatial correlation of fluctuations in the microscopic number densities of moving particles and scatterers for a Lorentz gas subject to a temperature gradient, a concentration gradient, and an external electric field.

There has been considerable interest in the study of spatial correlations in nonequilibrium systems. It was stimulated by the fact that because of the gradients in the system, such correlation functions typically have a much longer range in space than their equilibrium counterparts. Among others, systems of fluids have been investigated both theoretically and experimentally [1]. For a dilute gas, the equal time correlation functions exhibit an algebraic decay in the nonequilibrium steady state, which was noticed by Ludwig [2] in 1962, but was first appreciated and fully investigated by Onuki [3], Ronis *et al.* [4], Kirkpatrick *et al.* [5], and Tremblay *et al.* [6], and the same is true for a fluid of arbitrary density [7]. These theoretical predictions of long range correlations for a fluid have been confirmed by light-scattering experiments [1,8].

It is also well known that the velocity autocorrelation function for a fluid in equilibrium shows an asymptotic long-time decay proportional to  $t^{-d/2}$ . This was first discovered by Alder and Wainwright [9] in a molecular-dynamics computer experiment for a system of hard spheres. This result came as a big surprise and motivated considerable theoretical work (see, for example, [10,11]). It was realized that these two seemingly different phenomena, namely long-time tails and long range correlations,

are closely related each other; their origins are the same hydrodynamic mode-coupling effects [1,7].

In contrast, for a Lorentz gas [12], where the medium is composed of fixed hard sphere scatterers, the velocity autocorrelation function in equilibrium was found to decay asymptotically for long times as  $t^{-(d/2+1)}$  [13]. This long-time tail was first explained in terms of correlated collision events [13] and later by means of phenomenological mode-coupling theory [14]. Having seen the close connection between long-time tail and long range correlations for a case of fluids, it is natural to expect that the long range correlation also exists for a Lorentz gas. However, no such correlations have been reported. This situation motivated us to search for possible long range correlations for a Lorentz gas in some nonequilibrium states. Technically our analysis is based on a kinetic equation for the two-point correlation function. In general, we find long range correlations do exist but they are somewhat weaker (faster decaying) than in the fluid case, just as the long-time tail in the Lorentz gas is weaker. This last point suggests it would be interesting to study correlations in a nonequilibrium quantum Lorentz gas, cf. point (4) in Sec. IV.

### II. BOGOLIUBOV-BORN-GREEN-KIRKWOOD-YVON HIERARCHY FOR A LORENTZ GAS

In a classical Lorentz gas, there is no interaction between moving particles, so that there is no mechanism to develop correlations between moving particles. A correlation can appear only between moving particles and scatterers. We therefore consider the following many body distribution functions:

$$\begin{aligned}
 C^{(l+1)}(x, \mathbf{R}^l, t) &= \frac{1}{V^{N_i}} \int dx^N d\mathbf{R}^{N_i} \sum_{i=1}^N \delta(x - x_i) \sum_{j_1=1}^{N_i} \delta(\mathbf{R} - \mathbf{R}_{j_1}) \times \dots \times \sum_{j_l=1}^{N_i} \delta(\mathbf{R}^l - \mathbf{R}_{j_l}) F(x^N, \mathbf{R}^{N_i}, t) \\
 &= \frac{N}{V^{N_i}} \frac{N_i!}{(N_i - l)!} \int dx^{N-l} d\mathbf{R}^{N_i-l} F(x^N, \mathbf{R}^{N_i}, t) \quad (j_1 \neq j_2 \neq \dots \neq j_l)
 \end{aligned}
 \tag{1}$$

$$h^{(l)}(\mathbf{R}^l) = \frac{1}{N} \int dx C^{(l+1)}(x, \mathbf{R}^l, t) = \frac{1}{V^l} \frac{N_i!}{(N_i - l)!}, \quad (2)$$

$$f(x, t) = C^{(1)}(x, \mathbf{R}^0, t) = \frac{N}{V N_i} \int dx^{N-1} d\mathbf{R}^{N_i} F(x^N, \mathbf{R}^{N_i}, t), \quad (3)$$

where,  $F(x^N, \mathbf{R}^{N_i}, t)$  is the classical distribution function of  $N$  moving particles for a fixed distribution of  $N_i$  scatterers;  $\mathbf{R}_i$  is the position of the  $i$ th scatterer,  $x = (\mathbf{v}, \mathbf{r})$ ; and  $V$  is the volume of system. The time dependence of  $h^{(l)}(\mathbf{R}^l)$  is dropped since the scatterers are fixed for each realization of disorder, and excluded volume effects have been ignored in the integration in terms of  $\mathbf{R}$ . Note that  $f(x, t)$  is the disorder-averaged one-particle distribution function.

We can construct a hierarchy of equations for the above correlation functions. In the presence of an electric field, the particles are assumed to be charged; however, interactions between particles are still ignored. The first two equations of the hierarchy are

$$\left\{ \partial_t + \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{r}} - \mathbf{E} \cdot \frac{\partial}{\partial \mathbf{v}} \right\} f(x, t) = \int d\mathbf{R} \hat{T}(x, \mathbf{R}) C^{(2)}(x, \mathbf{R}, t) \quad (4)$$

$$\left\{ \partial_t + \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{r}} - \hat{T}(x, \mathbf{R}) - \mathbf{E} \cdot \frac{\partial}{\partial \mathbf{v}} \right\} C^{(2)}(x, \mathbf{R}, t) = \int d\mathbf{R}' \hat{T}(x, \mathbf{R}') C^{(3)}(x, \mathbf{R}, \mathbf{R}', t), \quad (5)$$

where  $\hat{T}(x, \mathbf{R}')$  is the binary collision operator between a particle and a hard sphere scatterer given by [15],

$$\hat{T}(x, \mathbf{R}) = \delta(\mathbf{r} - \mathbf{R}) \sigma^2 \int_{\mathbf{v} \cdot \hat{\sigma} > 0} d\hat{\sigma} |\mathbf{v} \cdot \hat{\sigma}| [b_{\hat{\sigma}} - 1]. \quad (6)$$

Here,  $\sigma$  is the diameter of a scatterer, and  $\hat{\sigma}$  a unit vector that characterizes the geometry of the binary collision between the particle and the scatterer at contact. The operator  $b_{\hat{\sigma}}$  replaces the velocity  $\mathbf{v}$  of the particle before the collision by the one after the collision;  $\mathbf{v}^* = \mathbf{v} - 2\hat{\sigma}(\mathbf{v} \cdot \hat{\sigma})$ . In giving Eq. (6) the difference in position between the particle and scatterer is neglected. It is easily shown that this is a low scatterer density approximation. We choose units such that the mass of the moving particle is 1,  $e = 1$ , and  $k_B = 1$ . In order to proceed further, we closely follow the paper [16] and use the cluster expansion of  $C^{(3)}(x, \mathbf{R}, \mathbf{R}')$ , etc., yielding

$$\begin{aligned} C^{(2)}(x, \mathbf{R}, t) &= f(x, t) h^{(1)}(\mathbf{R}) + \bar{C}^{(2)}(x, \mathbf{R}, t) \\ C^{(3)}(x, \mathbf{R}, \mathbf{R}') &= f(x, t) h^{(1)}(\mathbf{R}) h^{(1)}(\mathbf{R}') \\ &\quad + \bar{C}^{(2)}(x, \mathbf{R}, t) h^{(1)}(\mathbf{R}') \\ &\quad + \bar{C}^{(2)}(x, \mathbf{R}', t) h^{(1)}(\mathbf{R}) \\ &\quad + \bar{C}^{(3)}(x, \mathbf{R}, \mathbf{R}', t), \end{aligned} \quad (7)$$

where  $\bar{C}^{(2)}(x, \mathbf{R}, t)$  and  $\bar{C}^{(3)}(x, \mathbf{R}, \mathbf{R}', t)$  are defined through these formulas. By substituting (7) into Eqs. (4) and (5), we have

$$\begin{aligned} \left\{ \partial_t + \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{r}} - \int d\mathbf{R} \hat{T}(x, \mathbf{R}) h^{(1)}(\mathbf{R}) - \mathbf{E} \cdot \frac{\partial}{\partial \mathbf{v}} \right\} f(x, t) &= \int d\mathbf{R} \hat{T}(x, \mathbf{R}) \bar{C}^{(2)}(x, \mathbf{R}, t) \\ \left\{ \partial_t + \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{r}} - \int d\mathbf{R}' \hat{T}(x, \mathbf{R}') h^{(1)}(\mathbf{R}') - \mathbf{E} \cdot \frac{\partial}{\partial \mathbf{v}} \right\} \bar{C}^{(2)}(x, \mathbf{R}, t) \\ &= \hat{T}(x, \mathbf{R}) \left\{ f(x, t) h^{(1)}(\mathbf{R}) + \bar{C}^{(2)}(x, \mathbf{R}, t) \right\} + \int d\mathbf{R}' \hat{T}(x, \mathbf{R}') \bar{C}^{(3)}(x, \mathbf{R}, \mathbf{R}', t). \end{aligned} \quad (8)$$

Now, we make the ansatz that the right-hand side of the first equation and the last two terms of the second equation can be ignored compared to other terms in a dilute scatterer approximation [16]. Then we have,

$$\left\{ \partial_t + \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{r}} - n_i \hat{T}(\mathbf{v}) - \mathbf{E} \cdot \frac{\partial}{\partial \mathbf{v}} \right\} f(x, t) = 0 \quad (9)$$

$$\left\{ \partial_t + \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{r}} - n_i \hat{T}(\mathbf{v}) - \mathbf{E} \cdot \frac{\partial}{\partial \mathbf{v}} \right\} \bar{C}^{(2)}(x, \mathbf{R}, t) = n_i \hat{T}(x, \mathbf{R}) f(x, t), \quad (10)$$

where  $n_i = N_i/V$  and  $\hat{T}(\mathbf{v})$  is defined by  $\delta(\mathbf{r} - \mathbf{R}) \hat{T}(\mathbf{v}) \equiv \hat{T}(x, \mathbf{R})$ .

Equation (9) is the Boltzmann equation for a Lorentz gas and Eq. (10) is an approximate equation for the two-point correlation function. These are closed equations for  $\bar{C}^{(2)}(x, \mathbf{R}, t)$ ; i.e., by means of Eq. (9), we can solve for  $f(x, t)$ , then  $\bar{C}^{(2)}(x, \mathbf{R}, t)$  is determined by Eq. (10). It is easily seen at this stage that the binary collision operator  $\hat{T}(x, \mathbf{R})$  just changes the direction of  $\mathbf{v}$ , so that for any function  $f(v)$  of  $v$  only,  $\hat{T}(x, \mathbf{R}) f(v) = 0$ ; the left-hand side of (10) is nonzero only when  $f(x, t)$  has anisotropy

in velocity space, namely in nonequilibrium.

Let us consider the nonequilibrium steady state in which an external electric field, temperature gradient, and chemical potential gradient are present. Up to the first order in those mechanical and thermal perturbations, the steady state solution of Eq. (9) is given by [17]

$$f(x) = f_{le} + \delta f \\ = f_{le} + \frac{\tau_v f_0}{T} \left\{ -(\mathbf{E} + \nabla\mu) + \frac{(\mu - \epsilon)}{T} \nabla T \right\} \cdot \mathbf{v}. \quad (11)$$

Here,  $f_{le}$  is the local equilibrium distribution function,  $e^{-\beta(x)[\epsilon(v) - \mu(x)]}$ ;  $f_0$  is the equilibrium distribution function,  $e^{-\beta(\epsilon(v) - \mu)}$ ,  $\tau_v = 1/(n_i \pi \sigma^2 v)$ ;  $\beta(x)$  and  $\mu(x)$  are space dependent inverse temperature and chemical potential, respectively; and  $\beta$  and  $\mu$  are the corresponding mean values.

Although this is a well known result, it involves more than just the solution of Eq. (9). In order to obtain the solution (11), the local equilibrium assumption was made [18]; however, there is no energy dissipation mechanism between moving particles nor between the particle and scatterer in Eq. (9). Accordingly, the electric field accelerates particles indefinitely, which leads to Joule heating, or in the absence of the electric field, a particle that acquires the kinetic energy from one boundary wall will possess the same energy until it reaches another wall and cannot relax into the local state determined by the local temperature and density. Therefore, on using Eq. (11) as our starting point, some energy transfer mechanism has to be implicitly assumed. In this paper we want to focus on the effect of elastic collisions on correlation functions. Thus, we will study Eq. (10) without any modification that would appear from the energy relaxation mechanisms. Physically this is a good approximation if the elastic scattering rate is large compared to the inelastic one.

### III. EIGENVALUE ANALYSIS OF THE KINETIC EQUATIONS

We solve Eq. (10) by means of an eigenvalue analysis. Introducing the Fourier transformation for  $\mathbf{r}$ , Eq. (10) in the steady state can be written as

$$\left\{ i\mathbf{k} \cdot \mathbf{v} - n_i \hat{T}(\mathbf{v}) - \mathbf{E} \cdot \frac{\partial}{\partial \mathbf{v}} \right\} \bar{C}^{(2)}(\mathbf{k}, \mathbf{v}, \mathbf{k}') \\ = n_i \hat{T}(\mathbf{v}) \delta f(\mathbf{v}) (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}'). \quad (12)$$

Now we look for the eigenvalue and the eigenfunctions of operator  $i\mathbf{k} \cdot \mathbf{v} - n_i \hat{T}(\mathbf{v}) \equiv L_d$ . This operator acts on any function of  $\hat{\mathbf{v}}$ , but not of  $v$ , so that the function space

is defined for each  $v$ . We are interested in the diffusion mode, namely the eigenvalue that vanishes as  $k \rightarrow 0$ . For small  $k$ , the eigenfunction and the eigenvalue are

$$L_d |\Theta\rangle = \omega |\Theta\rangle, \\ |\Theta\rangle = 1 - i\tau_v \mathbf{k} \cdot \mathbf{v} + \dots, \\ \omega = D_v k^2 + \dots, \quad (13)$$

where  $D_v = v^2 \tau_v / 3 = v / (3n_i \pi \sigma^2)$ . The  $v$  dependence of diffusion constant  $D_v$  and relaxation time  $\tau_v$  simply reflects the fact that in a Lorentz gas one particular trajectory of a moving particle is realized by a class of initial conditions that have the same directions in velocities and initial positions but different speeds,  $v$ 's, and the times the particles spend in this trajectory are scaled by the ratio between those  $v$ 's. This consideration also leads us to define the inner product in the function space on which  $L_d$  acts as follows:

$$\langle \phi(\mathbf{k}, \mathbf{v}) | \varphi(\mathbf{k}, \mathbf{v}) \rangle_{in} = \int \frac{d\hat{\mathbf{v}}}{S^d} \phi^*(\mathbf{k}, \mathbf{v}) \varphi(\mathbf{k}, \mathbf{v}), \quad (14)$$

where  $S^d$  is the solid angle element in the  $d$  dimension. Note that  $|\Theta\rangle$  is normalized to 1 in the limit of  $k \rightarrow 0$  by this inner product.

#### A. $\mathbf{E} = 0$ case

We first calculate the number density correlation function of particles and scatterers in the case of  $\mathbf{E} = 0$ . This correlation function is defined by

$$\langle \tilde{n}(\mathbf{r}) \tilde{N}_i(\mathbf{R}) \rangle = \langle n(\mathbf{r}) N_i(\mathbf{R}) \rangle - \langle n(\mathbf{r}) \rangle \langle N_i(\mathbf{R}) \rangle \\ \equiv \frac{1}{(2\pi)^3} \int d^3\mathbf{k} e^{i\mathbf{k} \cdot (\mathbf{r} - \mathbf{R})} D(\mathbf{k}), \quad (15)$$

$$(2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') D(\mathbf{k}) \\ \equiv \int d^3\mathbf{v} \bar{C}^{(2)}(\mathbf{k}, \mathbf{v}, \mathbf{k}', t) \\ = 4\pi \int_0^\infty v^2 dv \langle 1 | \bar{C}^{(2)} \\ \times (\mathbf{k}, \mathbf{v}, \mathbf{k}', t) | 1 \rangle_{in},$$

where  $n(\mathbf{r})$  is the density of particles,  $N_i(\mathbf{R})$  is the density of scatterers, and  $|1\rangle = 1$ . When there is no electric field, Eq. (12) can be formally solved as

$$\bar{C}^{(2)}(\mathbf{k}, \mathbf{v}, \mathbf{k}', t) = \frac{1}{L_d} n_i \hat{T}(\mathbf{v}) \delta f(\mathbf{v}) (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}'). \quad (16)$$

It is obvious that the diffusion mode  $|\Theta\rangle$  is the dominant part of the eigenvalue spectrum of  $L_d$  for  $k \rightarrow 0$ , or for long distances. Inserting the projection operator  $|\Theta\rangle\langle\Theta|$ ,  $D(\mathbf{k})$  for  $k \rightarrow 0$  is given by

$$D(\mathbf{k}) = 4\pi \int_0^\infty v^2 dv \left\langle 1 \left| \frac{1}{L_d} \right| \Theta \right\rangle_{in} \langle \Theta | n_i \hat{T}(\mathbf{v}) \delta f(\mathbf{v}) | 1 \rangle_{in} \\ = 4\pi \int_0^\infty v^2 dv \frac{1}{D_v k^2} \int \frac{d\hat{\mathbf{v}}}{S^d} [1 + i\tau_v (\mathbf{k} \cdot \mathbf{v})] \left[ \mathbf{v} \cdot \left\{ \nabla\mu - \frac{\mu - \epsilon}{T} \nabla T \right\} \right] \frac{f_0}{T} \\ \cong \frac{N}{V} \frac{1}{T} \left\{ \frac{i\mathbf{k} \cdot \nabla\mu}{k^2} - \left( \frac{\mu}{T} - \frac{3}{2} \right) \frac{i\mathbf{k} \cdot \nabla T}{k^2} \right\}. \quad (17)$$

In the real space this implies the algebraic decay

$$\langle \bar{n}(\mathbf{r}) \tilde{N}(\mathbf{R}) \rangle = -\frac{1}{4\pi^2} \frac{N}{VT} \frac{1}{r^2} \left\{ \hat{\mathbf{r}} \cdot \nabla \mu - \left( \frac{\mu}{T} - \frac{3}{2} \right) \hat{\mathbf{r}} \cdot \nabla T \right\}$$

in three dimensions. This shows that the correlations are of *long range* and that they decay as  $r^{-2}$ . As seen from the above derivation, this is due to the coupling between the diffusion mode and the constraints imposed to maintain the nonequilibrium steady state.

#### B. $\mathbf{E} \neq \mathbf{0}$ case

We next consider the  $\mathbf{E} \neq \mathbf{0}$  and the  $\nabla \mu = \nabla T = \mathbf{0}$  case. The existence of an additional term  $\mathbf{E} \cdot \partial/\partial \mathbf{v}$  in Eq. (12) allows us to investigate  $D(\mathbf{k})$  not only in the first order in  $E$  but also to arbitrary strength in  $E$ . The operator  $\mathbf{E} \cdot \partial/\partial \mathbf{v}$  is defined in a different space from that of  $L_d$ . However, the same eigenfunction  $|\Theta\rangle = 1 - i\tau_v \mathbf{k} \cdot \mathbf{v}$  solves the eigenvalue equation for  $L_d - \mathbf{E} \cdot \partial/\partial \mathbf{v}$  with the new eigenvalue  $\omega = D_v k^2 + i\tau_v \mathbf{k} \cdot \mathbf{E}$ . This is true up to second order in  $k$  for the term independent of  $E$  and to first order in  $k$  for the term that depends on  $E$ . Thus with this approximation we can regard this operator as one that belongs in the same operator space as  $L_d$ . Therefore the equation for  $D(\mathbf{k})$  can be solved in the same way as before,

$$\begin{aligned} D(\mathbf{k}) &= 4\pi \int_0^\infty v^2 dv \left\langle 1 \left| \frac{1}{L_d - \mathbf{E} \cdot \partial_v} \right| \Theta \right\rangle_{in} \langle \Theta | n_i \hat{T}(\mathbf{v}) \delta f(\mathbf{v}) | 1 \rangle_{in} \\ &= 4\pi \int_0^\infty v^2 dv \frac{1}{D_v k^2 + i\tau_v \mathbf{k} \cdot \mathbf{E}} \int \frac{d\hat{\mathbf{v}}}{S_d} [1 + i\tau_v (\mathbf{k} \cdot \mathbf{v})] (\mathbf{v} \cdot \mathbf{E}) \frac{f_0}{T} \\ &\doteq 4\pi \int_0^\infty v^2 dv \frac{1}{3} \frac{v^2 i\tau_v (\mathbf{k} \cdot \mathbf{E})}{D_v k^2 + i\tau_v (\mathbf{k} \cdot \mathbf{E})} \frac{f_0}{T}. \end{aligned} \quad (18)$$

Integrating over  $v$  yields

$$\begin{aligned} D(\mathbf{k}) &= \frac{N}{V} \frac{i\mathbf{k} \cdot \mathbf{E}}{k^2 T} - 3 \frac{N}{V} \left( \frac{i\mathbf{k} \cdot \mathbf{E}}{k^2 T} \right)^2 + \frac{N}{V} \sqrt{\frac{27\pi}{2}} \left( \frac{i\mathbf{k} \cdot \mathbf{E}}{k^2 T} \right)^{\frac{5}{2}} \exp \left[ \frac{3 i\mathbf{k} \cdot \mathbf{E}}{2 k^2 T} \right] \left\{ 1 - \text{Erf} \left[ \sqrt{\frac{3 i\mathbf{k} \cdot \mathbf{E}}{2 k^2 T}} \right] \right\} \\ &\doteq \frac{N}{V} \frac{i\mathbf{k} \cdot \mathbf{E}}{k^2 T} - 3 \frac{N}{V} \left( \frac{i\mathbf{k} \cdot \mathbf{E}}{k^2 T} \right)^2 + \frac{N}{V} \sqrt{\frac{27\pi}{2}} \left( \frac{i\mathbf{k} \cdot \mathbf{E}}{k^2 T} \right)^{\frac{5}{2}} + \dots, \end{aligned} \quad (19)$$

where the last line was expanded in powers of  $(i\mathbf{k} \cdot \mathbf{E})/(k^2 T)$ . This is compatible with the above approximation and is valid as long as the electric potential difference between the two boundary walls is sufficiently small compared to the mean temperature of the system,

$$\frac{\mathbf{k} \cdot \mathbf{E}}{k^2 T} \ll 1 \implies r \sim \frac{1}{k} \ll \frac{LT}{\Phi_1 - \Phi_2},$$

where  $\Phi_i (i = 1, 2)$  are the electric scalar potentials at the boundary walls and  $L$  is the length of the system.

The result, Eq. (19), indicates the long range correlations. In particular, the first term is the same as the term involving the chemical potential gradient in Eq. (17). It also shows that the higher order terms in  $E$  have stronger singularities in  $k$  space, and that the expansion in powers of  $E$  is nonanalytic.

In principle, when we go beyond first order in  $E$ , the higher order corrections to  $f(x, t)$  must also be taken into account. Those corrections, however, involve the consideration of the dynamics of scatterers, Joule heating, etc., and require additional parameters such as the inelastic scattering rate discussed at the end of the last section. Our results are due solely to the coupling between the electric field and the diffusion mode that originates from

elastic scattering and are therefore independent of these corrections.

#### IV. CONCLUSION AND DISCUSSION

We have shown that the correlation function of the number density of particles and scatterers decays algebraically for a Lorentz gas in a nonequilibrium steady state. To first order in a concentration gradient, a temperature gradient and an electric field, it decays asymptotically as  $1/r^2$ , while it has stronger correlation at higher order in the electric field. Those long range correlations are due to the coupling between diffusive mode and thermal or mechanical perturbations that maintain the nonequilibrium steady state. Our method was based on a simple analysis of the kinetic equation for the two-particle correlation function.

We conclude this paper with a number of remarks:

(i) Our method is considerably simpler than that for a fluid [7]. Due to the lack of momentum conservation in collision process in a Lorentz gas, there are only two collisional invariances,  $1$  and  $v^2$ , which correspond to number density and energy density conservation law, respec-

tively. Moreover, because of the absence of an energy relaxation mechanism, the particles having different  $v$ 's are never mixed up. The diffusion process occurs only involving particles having the same  $v$ 's. This means that  $v$  can be regarded as a parameter characterizing the class of particles, and accordingly there is only one collisional invariant: 1. This is the reason why we have a simple solution for the eigenvalue equation (13) and the definition of the inner product is given by Eq. (14). The statistical distribution of energies emerges only in the preparation of the local equilibrium. It is interesting to compare this situation with the case of the self-diffusion process in a fluid [19]. There, the collision operator in the Lorentz-Boltzmann equation involves the equilibrium distribution function  $f_0$ . Thus, the inner product also involves  $f_0$ , and the similar eigenvalue analysis yields the constant  $D$ . In this sense, the diffusion process in a Lorentz gas is special.

(ii) In wave number space, our results for correlation functions of particle density and scatterer density are summarized as follows:

$$D(\mathbf{k}) \sim \frac{i\hat{\mathbf{k}} \cdot (\text{gradient of } T, \mu, \text{ or } \Phi)}{kT} + \sim \frac{(\hat{\mathbf{k}} \cdot \mathbf{E})^2}{k^2 T^2} + \dots$$

In contrast, for a fluid in a temperature gradient, the density density correlation function has been shown to have a long range [7], which in wave number space is

$$\sim \frac{(i\hat{\mathbf{k}} \cdot \nabla T)}{k^2 T} + \sim \frac{(\hat{\mathbf{k}} \cdot \nabla T)^2}{k^4 T^2}.$$

This comparison shows us that the Lorentz gas correlations are less singular as  $k \rightarrow 0$  than those in a fluid. It is generally believed that the smaller the long-time tail exponent, the weaker the long range correlations. The long-time tail for a Lorentz gas is  $t^{-(d/2+1)}$ , which is also weaker than the  $t^{-d/2}$  for a fluid. Thus our results support this general scheme. In the case of fluids, the connection between long-time tails and long range correlations is well understood [1], [7]. We investigate this connection for a Lorentz gas elsewhere [20].

(iii) The spatial correlations in diffusive systems kept in a nonequilibrium steady state have been studied by many authors [1,21,22]. In general the particle-particle density correlation function is studied using fluctuating hydrodynamics, and long range spatial correlations are found. However, these are due to the interaction between particles and are therefore distinct from our study.

(iv) In this paper we studied a classical Lorentz gas. The quantum version of the problem has a long-time tail that decays as  $t^{-d/2}$  [23]. One therefore expects stronger long range correlations in nonequilibrium quantum Lorentz gases than in the classical Lorentz gas. In addition, even if there is no interaction between particles, there are correlations due to the Pauli exclusion principle. For example, the two-particle Wigner function cannot be simply factorized into two one-particle Wigner functions. These problems will be considered elsewhere [24].

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